Gauge Dependence in Chern-Simons Theory

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Abstract

We compute the contribution to the modulus of the one-loop effective action in pure non-Abelian Chern-Simons theory in an arbitrary covariant gauge. We find that the results are dependent on both the gauge parameter (α) and the metric required in the gauge fixing. A contribution arises that has not been previously encountered; it is of the form $(\alpha/\sqrt{p^2})\epsilon_{\mu\lambda\nu}p^\lambda$. This is possible as in three dimensions α is dimensionful. A variant of proper time regularization is used to render these integrals well behaved (although no divergences occur when the regularization is turned off at the end of the calculation). Since the original Lagrangian is unaltered in this approach, no symmetries of the classical theory are explicitly broken and $\epsilon_{\mu\lambda\nu}$ is handled unambiguously since the system is three dimensional at all stages of the calculation. The results are

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shown to be consistent with the so-called Nielsen identities which predict the explicit gauge parameter dependence using an extension of BRS symmetry. We demonstrate that this α dependence may potentially contribute to the vacuum expectation values of products of Wilson loops.

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I. INTRODUCTION

Non-Abelian Chern-Simons theory is an example of a three-dimensional topological theory. It has been argued that radiative corrections in the theory serve only to displace the coupling constant, despite the fact that the gauge fixing term that must be appended to the classical Lagrangian is metric dependent [1]. This is consistent with a number of (but not all) perturbative calculations, each of which requires some form of regularization [2–9]. Regulating this model is complicated by the three dimensional nature of the tensor $\epsilon_{\alpha\beta\gamma}$ occurring in the initial Lagrangian, and by the fact that the theory is topological - it is difficult to respect these properties if one regulates by altering the initial Lagrangian in some way. (For example, addition of a Yang-Mills term to the Chern-Simons Lagrangian [3,5,8], while rendering the theory super-renormalizable, does not respect its topological nature and simple Pauli-Villars regularization [2] breaks the BRS invariance of the tree-level effective action. [6]) A way of circumventing this difficulty is to use a variant of operator regularization [10]; in this approach the initial Lagrangian is not modified - rather, the operators occurring in the closed form expression for the generating functional are regulated.

At one loop order, one encounters a functional determinant $\det M$. Since M is linear in the derivative operator, operator regularization cannot be applied directly; one must instead consider $\det^{\frac{1}{2}} M^2$. However these two expressions may not be equal - a phase associated with $\det M$ may be lost if M has negative eigenvalues [1,11,12]. This phase may be determined by considering the so-called η-function [13]. In [11] it is shown that the contribution of the η-function to $\det M$ is consistent with the expectations of [1], and that this result is independent of the gauge fixing parameter α. On the other hand, in [14] it has been demonstrated that if operator regularization is used to evaluate the contribution of the two-point function to $\det^{\frac{1}{2}} M^2$ (associated with modulus of $\det M$) then a result proportional to $(g_{\mu\nu} - p_\mu p_\nu/p^2)$ is obtained. This respects the gauge invariance of the theory, but clearly is not topological in character. This computation was performed in the so-called Landau-Honerkamp gauge.

In this paper, we generalize the computation of [14] by including a gauge parameter α (with the Landau gauge recovered in the $\alpha = 0$ limit). Operator regularization cannot be used as in [14] when $\alpha \neq 0$ due to the form of $\det^{\frac{1}{2}} M^2$; however a simple modification of the procedure (explained below) allows us to circumvent the difficulties which occur. It is found that the two-point contribution to $\det^{\frac{1}{2}} M^2$ is in fact dependent on α. The form of these terms has not been anticipated [8] as the gauge parameter α is dimensionful; they are proportional to $(\alpha^2/(p^2)^{3/2})(p^2 g_{\mu\nu} - p_\mu p_\nu)$ and $(\alpha/\sqrt{p^2})\epsilon_{\mu\lambda\nu}p^\lambda$. We now give the details of the calculation, demonstrate consistency with an extension of the BRS identities (known as the Nielsen identities), and then show that to one-loop order there may be non-vanishing contributions to the vacuum expectation value of a product of Wilson loops that are of order α.

II. THE TWO-POINT FUNCTION

Let us consider the classical Chern-Simons Lagrangian

$$\mathcal{L}_{cs} = \frac{1}{2}\epsilon_{\mu\lambda\nu}\left(V_\mu^a \partial_\lambda V_\nu^a + \frac{g}{3}f^{abc}V_\mu^a V_\lambda^b V_\nu^c\right). \tag{1}$$

If $g = \sqrt{\frac{4\pi}{k}}$ (for some integer k) then (1) describes a topological quantum field theory which is invariant under gauge transformations of arbitrary winding number. We add to it the gauge-fixing Lagrangian

$$\mathcal{L}_g = \frac{-1}{2\alpha}\left(D_\mu^{am}(A)Q_\mu^m\right)\left(D_\nu^{an}(A)Q_\nu^n\right) \tag{2}$$

where V has been split into the sum of classical background field A and a quantum field Q [15], and $D_\mu^{ab}(A) = \partial_\mu \delta^{ab} + g f^{apb} A_\mu^p$. (Note that in (2) α is dimensionful.) The ghost Lagrangian associated with (2) is

$$\mathcal{L}_{gh} = \overline{c}^a D_\mu^{ab}(A) D_\mu^{bc}(A+Q)c^c \ . \tag{3}$$

As this is independent of α, its contribution to the one-loop generating functional was computed in [14] and need not be considered further.

Usually the smooth limit to the Landau-Honerkamp gauge requires that a first order gauge fixing formalism be used: i.e. (2) being replaced by

$$\mathcal{L}_g = B^a D_\mu^{ab}(A)Q_\mu^b + \frac{\alpha}{2}B^a B^a \ . \tag{4}$$

The Landau-Honerkamp gauge is obtained now by simply setting α to zero (as was done in [1]). However, when we retain α non-zero throughout the calculations, the gauge fixing terms (4) make the computations prohibitively difficult. For this reason we choose to work with the gauge fixing Lagrangian (2).

The contribution to the one-loop generating functional coming from (1) and (2) is given by

$$\Gamma^{(1)} = \ln \ \det^{-\frac{1}{2}}(M_{\mu\nu}^{ab}). \tag{5}$$

Unfortunately, the factor

$$M_{\mu\nu}^{ab} = \epsilon_{\mu\lambda\nu}D_\lambda^{ab}(A) + \frac{1}{\alpha}D_\mu^{am}(A)D_\nu^{mb}(A) \tag{6}$$

is not in a form that is appropriate for applying operator regularization [10,12] directly to (5). The appropriate procedure in this case [12] is to examine separately the modulus and phase of $\Gamma^{(1)}$. The phase calculation, for arbitrary α has been computed elsewhere [11]. We now concentrate on the contribution from the modulus, i.e. we examine

$$| \Gamma^{(1)} | \ = \ \ln \det^{-\frac{1}{4}}\left(M_{\mu\kappa}^{ap}M_{\kappa\nu}^{pb}\right) \tag{7a}$$

$$= -\frac{1}{4}\text{tr} \ln\left(M_{\mu\kappa}^{ap}M_{\kappa\nu}^{pb}\right) \tag{7b}$$

$$= \frac{1}{4}\int_0^\infty \frac{dt}{t}\text{tr}\left(e^{-M^2 t}\right) \ . \tag{7c}$$

A perturbative expansion of $| \Gamma^{(1)} |$ can be developed if we first partition M^2 into the sum $M_0^2 + M_1^2$ where M_1^2 contains all the dependence on the background field A_μ^a, and secondly use the Schwinger expansion [10,16]

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$$\operatorname{tr} e^{-M^2 t} = \operatorname{tr}\Big[e^{-M_0^2 t} + (-t)e^{-M_0^2 t} M_1^2 \tag{8}$$

$$+ \frac{(-t)^2}{2} \int_0^1 du \ e^{-(1-u)M_0^2 t} M_1^2 e^{-u M_0^2 t} M_1^2 + ...\Big]$$

As we are interested in the two point function we need only retain terms which are second order in the background field A_μ^a.

From (6) we see that

$$(M_0^2)_{\mu\nu}^{ab} = \delta^{ab}\Big[p^2 \delta_{\mu\nu} - p_\mu p_\nu + \frac{1}{\alpha^2} p^2 p_\mu p_\nu\Big] \ , \ (p \equiv -i\partial) \tag{9}$$

so that

$$e^{-M_0^2 t} = e^{-p^2 t} T_{\mu\nu} + e^{-\frac{p^4 t}{\alpha^2}} L_{\mu\nu} \tag{10}$$

where $T_{\mu\nu} \equiv \delta_{\mu\nu} - p_\mu p_\nu / p^2$, and $L_{\mu\nu} \equiv p_\mu p_\nu / p^2$.

(In deriving (10), we have used that fact that T and L constitute a complete set of orthogonal projection operators.)

The presence of the factor p^4 in the second exponential in (10) makes implementation of operator regularization as outlined in [10] prohibitively difficult. We can however make use of the formula

$$\frac{1}{a^m b^n} \int_0^\infty dt \ t \int_0^1 du \ e^{-[\frac{a^p}{A}(1-u) + \frac{b^q}{B} u]t} \tag{11}$$

$$= \frac{AB}{\Gamma(m+p)\Gamma(n+q)} \int_0^1 dx \ x^{m+p-1}(1-x)^{n+q-1} \int_0^\infty d\tau \ \tau^{m+n+p+q-1} e^{-[ax+b(1-x)]\tau}$$

in order to eliminate dependence on p^4 that may occur in the arguments of exponentials appearing in (8). In order to regulate any potential divergences in integrals appearing at this stage in the calculation, the factor of $\tau^{m+n+p+q-1}$ in (8) is replaced by $\tau^{\lambda+m+n+p+q-1}$; the regulating parameter λ is allowed to pass to the limit zero at the end of the calculation. In the present situation, no divergence arises in the final expression in this $\lambda \to 0$ limit, so in principle the parameter λ is superfluous, although it does serve to render all intermediate expressions in the calculation well defined.

In (11) we take $p+m$ and $q+n$ to be greater than zero, otherwise we would just be left with tadpole integrals of the form

$$\int d^3 p \frac{1}{(p^2)^a} \ , \ (a > 0)$$

which are taken to be zero [14].

We now outline the steps used to obtain the contribution from $| \Gamma^{(1)} |$ to the two-point function to $\det^{-\frac{1}{4}} M^2$. First, upon making use of the expansion (8) in (7c) with operator M defined in (6) we obtain

$$| \Gamma_{AA}^{(1)} | = g^2 C_2 \int_0^\infty dt \ t \int_0^1 du \int \frac{d^3 p \ d^3 q}{(2\pi)^3} \Big\{ e^{-[p^2(1-u) + q^2 u]t}$$

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$$\left[-\frac{(q\cdot p)^2}{q^2}A(+)\cdot A(-) + \Big(q\cdot A(+)p\cdot A(-) + q\cdot A(-)p\cdot A(+)\Big)(1+\frac{q\cdot p}{q^2}) \right]$$

$$+e^{-[q^2(1-u)+\frac{p^4}{\alpha^2}u]t}\left[A(+)\cdot A(-)\frac{(q\cdot p)^2}{p^2} + q\cdot A(+)q\cdot A(-) \right.$$

$$\left. -\frac{q\cdot p}{p^2}\Big(q\cdot A(+)p\cdot A(-) + q\cdot A(-)p\cdot A(+)\Big) \right. \tag{12}$$

$$+\frac{i}{\alpha}\Big(q\cdot A(-)q\cdot p\times A(+) - q\cdot A(+)q\cdot p\times A(-) + 2q\cdot p(q-p)\cdot A(+)\times A(-)\Big)$$

$$+\frac{p^2}{\alpha^2}\Big(-4q\cdot pA(+)\cdot A(-) + 2q\cdot A(+)p\cdot A(-) + 2q\cdot A(-)p\cdot A(+)$$

$$\left. -p\cdot A(-)p\cdot A(+) - q\cdot A(-)q\cdot A(+) - \frac{1}{q^2}q\cdot p\times A(+)q\cdot p\times A(-)\Big) \right]$$

$$+e^{-[q^4(1-u)+p^4u]t/\alpha^2}\left[\frac{1}{\alpha^2}(1+\frac{p^2}{q^2})q\cdot p\times A(+)q\cdot p\times A(-) \right.$$

$$+\frac{i}{\alpha^3}q\cdot p\times A(+)\Big(q^2p\cdot A(-) + p^2q\cdot A(-) + (p^2+q^2)(q+p)\cdot A(-)\Big)$$

$$\left.\left. +\frac{1}{\alpha^4}p^2q^2\Big(p\cdot A(-)p\cdot A(+) + q\cdot A(+)p\cdot A(-) + q\cdot A(-)p\cdot A(+)\Big) \right]\right\} .$$

In arriving at (12), we have computed functional traces in momentum space [10,16] so that

$$(2\pi)^{3/2}\langle p\mid A_\mu^a\mid q\rangle = A_\mu^a(p-q) \ . \tag{13}$$

We have used the notation $f^{amn}f^{bmn} = C_2\delta^{ab}$, $A_\mu(\pm) \equiv A_\mu^a(\pm(q-p))$ and $X\cdot Y\times Z \equiv \epsilon_{\alpha\beta\gamma}X_\alpha Y_\beta Z_\gamma$. All contributions to the two-point function which are eventually proportional to tadpole integrals have been dropped in (12).

By using (11), (12) can be converted into a sum of integrals in which the argument of the exponential occurring in each integrand is just $[xp^2 + (1-x)q^2]\tau$, allowing us to use the standard integrals to compute $\Gamma_{AA}^{(1)}$, as was done in the examples given in [10]. Some tedious calculation leaves us with the result

$$|\Gamma_{AA}^{(1)}| = g^2\frac{C_2}{32}\int d^3p\sqrt{p^2}\left\{ \Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big)\Big(1+\frac{\alpha^2}{4p^2}\Big) + \frac{i\alpha}{p^2}\epsilon_{\mu\lambda\nu}p_\lambda \right\}A_\mu^a(p)A_\nu^a(-p) \ . \tag{14}$$

This is transverse, as is required by gauge invariance, but is manifestly dependent on both the metric and gauge parameter α. Neither of the terms dependent on α have been anticipated [8]. The dependence on α occurs in the transverse direction in (14), which is unlike what occurs in Yang-Mills theory where gauge-parameter dependence is confined to the unphysical longitudinal sector of the vector field. This does not imply that the propagation of physical degrees of freedom depends on an arbitrary parameter, as the pure Chern-Simons theory in fact has no dynamical content!

III. NIELSEN IDENTITIES

We now turn to the Nielsen identities to test the consistency of our expression for $\Gamma_{AA}^{(1)}$. Much as the Ward-Takahashi and Slavnov-Taylor identities arise from considering the BRS symmetry of the Lagrangian, one obtains the Nielsen Identities [17,18] by extending this symmetry to include variations in the gauge parameter. In doing so we are able to obtain relations between the variation of the gauge parameter in a given Green's function with their product of other Green's functions (e.g. $\frac{\partial}{\partial \alpha}\Gamma_1 = \Gamma_2\Gamma_3$). The relevant Nielsen identity for $\Gamma_{AA}^{(1)}$ will be derived and the other Green's functions arising in the identity will be calculated. We will then show that the α-dependence in (14) is consistent with this identity.

In the background field formalism the full Lagrangian for a non-Abelian gauge field (including all sources) [18] is given by

$$\mathcal{L} = \mathcal{L}_{cs}(A+Q) - \frac{1}{2\alpha}\left(D_\mu(A)Q_\mu\right)\left(D_\nu(A)Q_\nu\right) + \overline{c}D_\mu(A)D_\mu(A+Q)c$$
$$+ K_\mu D_\mu\left(A+Q\right)c + \frac{g}{2}K\cdot c\times c + L_\mu\left(D_\mu(A+Q)\overline{c} - K_\mu\right), \tag{15}$$

where $K\cdot c\times c = K^a f^{abc}c^b c^c$ and other group indices have been suppressed. The Lagrangian is invariant under the usual BRS transformations:

$$\delta A_\mu = -L_\mu\lambda \qquad\qquad \delta Q_\mu = D_\mu(A+Q)c\lambda + L_\mu\lambda$$
$$\delta c = \frac{g}{2}c\times c\lambda \qquad\qquad \delta\overline{c} = \frac{-1}{\alpha}D_\mu(A)Q_\mu\lambda \tag{16}$$

where λ is a global Grassmann parameter. By adding the following term parametrized by the global Grassmann variable χ (which does not alter the dynamics) to the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \frac{\chi}{2\alpha}\overline{c}D_\mu(A)Q_\mu, \tag{17}$$

the new Lagrangian is invariant under the following extended set of BRS transformations [17,18]

$$\delta A_\mu = -L_\mu\lambda \qquad\qquad \delta Q_\mu = D_\mu(A+Q)c\lambda + L_\mu\lambda$$
$$\delta c = \frac{g}{2}c\times c\lambda \qquad\qquad \delta\overline{c} = \frac{-1}{\alpha}D_\mu(A)Q_\mu\lambda + \frac{\chi}{2\alpha}\overline{c}\lambda \tag{18}$$
$$\delta\alpha = \chi\lambda \qquad\qquad \delta\chi = 0\,.$$

The generating functional is

$$Z = \int [\mathcal{D}Q_\mu\,\mathcal{D}\overline{c}\,\mathcal{D}c]\,e^{-\int d^3x\left(\mathcal{L}+J_\mu Q_\mu+\overline{J}_c c+\overline{c}J_{\overline{c}}\right)}, \tag{19}$$

and the 1PI generating functional is

$$\Gamma[Q_\mu, c, \overline{c}, A_\mu, L_\mu, K_\mu, K, \alpha, \chi] \equiv W[J_\mu, \overline{J}_c, J_{\overline{c}}, A_\mu, L_\mu, K_\mu, K, \alpha, \chi]$$
$$- \int d^3x\left[J_\mu Q_\mu + \overline{J}_c c + \overline{c}J_{\overline{c}}\right] \tag{20}$$

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where $W \equiv -\ln Z$. Γ is invariant in the following manner

$$\delta\Gamma \equiv \delta Q_\mu \frac{\delta\Gamma}{\delta Q_\mu} + \delta c \frac{\delta\Gamma}{\delta c} + \delta\bar{c} \frac{\delta\Gamma}{\delta\bar{c}} + \delta A_\mu \frac{\delta\Gamma}{\delta A_\mu} + \delta\alpha \frac{\delta\Gamma}{\delta\alpha} \tag{21}$$
$$= 0\,.$$

Note that integration over coordinate space is understood. The quantum fields here are not the ones which originally appear in the Lagrangian but rather the Legendre transform fields (i.e. vacuum expectation values of the Lagrangian fields) which appear in the proper generating functional. Thus the BRS variations in terms of the these fields are not necessarily given by (18) but may be expressed in terms of derivatives with respect to the sources (this can be shown to hold for composite fields as well). Also, for the particular Green's functions we will want to obtain, we may choose to set $L_\mu = 0$ (i.e. $\delta A_\mu = 0$). Thus equation (21) becomes

$$0 = \frac{\delta\Gamma}{\delta K_\mu}\frac{\delta\Gamma}{\delta Q_\mu} - \frac{\delta\Gamma}{\delta K}\frac{\delta\Gamma}{\delta c} + \left[\frac{1}{\alpha}D_\mu(A)Q_\mu - \frac{\chi}{2\alpha}\bar{c}\right]\frac{\delta\Gamma}{\delta\bar{c}} + \chi\frac{\delta\Gamma}{\delta\alpha}\,; \tag{22}$$

differentiating with respect to χ and then setting χ equal to 0 we obtain

$$0 = \frac{\delta^2\Gamma}{\delta\chi\delta K_\mu}\frac{\delta\Gamma}{\delta Q_\mu} - \frac{\delta\Gamma}{\delta K_\mu}\frac{\delta^2\Gamma}{\delta\chi\delta Q_\mu} - \frac{\delta^2\Gamma}{\delta\chi\delta K}\frac{\delta\Gamma}{\delta c} - \frac{\delta\Gamma}{\delta K}\frac{\delta^2\Gamma}{\delta\chi\delta c} + \frac{1}{\alpha}D_\mu(A)Q_\mu\frac{\delta^2\Gamma}{\delta\chi\delta\bar{c}} - \frac{1}{2\alpha}\bar{c}\frac{\delta\Gamma}{\delta\bar{c}} + \frac{\delta\Gamma}{\delta\alpha}\,. \tag{23}$$

Since we take no further derivatives with respect to the ghost fields or sources K we may set them to zero. Ghost number conservation then implies that all but the following terms vanish,

$$0 = \frac{\delta^2\Gamma}{\delta\chi\delta K_\mu}\frac{\delta\Gamma}{\delta Q_\mu} + \frac{1}{\alpha}D_\mu(A)Q_\mu\frac{\delta^2\Gamma}{\delta\chi\delta\bar{c}} + \frac{\delta\Gamma}{\delta\alpha}\,. \tag{24}$$

We may now set $Q_\mu = 0$ and differentiate the two remaining terms with respect to $A_\lambda(x)$ and $A_\nu(y)$ to obtain

$$0 = \frac{\delta^4\Gamma}{\delta\chi\delta K_\mu\delta A_\nu(y)\delta A_\lambda(x)}\frac{\delta\Gamma}{\delta Q_\mu} + \frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\lambda(x)}\frac{\delta^2\Gamma}{\delta A_\nu(y)\delta Q_\mu} + \frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\nu(y)}\frac{\delta^2\Gamma}{\delta A_\lambda(x)\delta Q_\mu}$$
$$+ \frac{\delta^2\Gamma}{\delta\chi\delta K_\mu}\frac{\delta^3\Gamma}{\delta A_\nu(y)\delta A_\lambda(x)\delta Q_\mu} + \frac{\delta^3\Gamma}{\delta A_\nu(y)\delta A_\lambda(x)\delta\alpha}\,. \tag{25}$$

There is a contribution from neither $\frac{\delta\Gamma}{\delta Q_\mu^a}$ nor from $\frac{\delta^2\Gamma}{\delta\chi\delta K_\mu^a}$ as these terms have a single un-contracted group index and Γ is invariant in group space. Thus we are left with the *Nielsen identity*

$$0 = \frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\lambda(x)}\frac{\delta^2\Gamma}{\delta A_\nu(y)\delta Q_\mu} + \frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\nu(y)}\frac{\delta^2\Gamma}{\delta A_\lambda(x)\delta Q_\mu} + \frac{\delta^3\Gamma}{\delta A_\nu(y)\delta A_\lambda(x)\delta\alpha}\,. \tag{26}$$

This equation is trivially satisfied at tree-level, since the proper vertex $\frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\lambda}$ have no tree-order contribution as is evident from (15). In fact, the identity (26) can be checked at

one-loop order without having to calculate radiative corrections to $\frac{\delta^2\Gamma}{\delta Q \delta A}$. We thus turn our attention to the calculation of one-loop contributions to $\frac{\delta^3\Gamma}{\delta\chi\delta K_\mu\delta A_\nu}$, pictured in figure 1. Power counting arguments show that these loop integrals are not divergent and so no regulating procedure will be required. The relevant terms in the extended Lagrangian bilinear in the quantum fields can be written as follows;

$$\mathcal{L}^{(2)} = \frac{1}{2}\left(Q_\mu^a \bar{c}^a c^a\right)\mathcal{M}_{\mu,\nu}^{ab}\begin{pmatrix} Q_\nu^b \\ \bar{c}^b \\ c^b \end{pmatrix} \tag{27}$$

with

$$\mathcal{M}_{\mu,\nu}^{ab} = \begin{bmatrix} \epsilon_{\mu\lambda\nu}D_\lambda^{ab}(A) + \frac{1}{\alpha}D_\mu^{am}(A)D_\nu^{mb}(A) & \frac{-1}{2\alpha}\chi D_\mu^{ab}(A) & f^{abc}K_\mu^c \\ \frac{-1}{2\alpha}\chi D_\nu^{ab}(A) & 0 & D_\lambda^{am}(A)D_\lambda^{mb}(A) \\ f^{abc}K_\nu^c & -D_\lambda^{am}(A)D_\lambda^{mb}(A) & 0 \end{bmatrix}. \tag{28}$$

Then the one loop effective action for K_μ, χ and A_μ is given by

$$\Gamma^{(1)} = \ln \operatorname{sdet}^{-\frac{1}{2}}\mathcal{M} \tag{29a}$$

$$= -\frac{1}{2}\operatorname{str}\ln\mathcal{M}. \tag{29b}$$

We split up $\mathcal{M}$ in the following manner

$$\ln\mathcal{M} = \ln\left(\mathcal{M}_O + \mathcal{M}_I\right) \tag{30}$$

where

$$\mathcal{M}_O = \begin{bmatrix} \epsilon_{\mu\lambda\nu}D_\lambda^{ab}(A) + \frac{1}{\alpha}D_\mu^{am}(A)D_\nu^{mb}(A) & 0 & 0 \\ 0 & 0 & D_\lambda^{am}(A)D_\lambda^{mb}(A) \\ 0 & -D_\lambda(A)^{am}D_\lambda^{mb}(A) & 0 \end{bmatrix} \tag{31}$$

and

$$\mathcal{M}_I = \begin{bmatrix} 0 & \frac{-1}{2\alpha}\chi D_\mu^{ab}(A) & f^{abc}K_\mu^c \\ \frac{-1}{2\alpha}\chi D_\nu^{ab}(A) & 0 & 0 \\ f^{abc}K_\nu^c & 0 & 0 \end{bmatrix}. \tag{32}$$

Writing (30) as

$$\ln(\mathcal{M}_O(1 + \mathcal{M}_O^{-1}\mathcal{M}_I)) = \ln\mathcal{M}_O + \ln(1 + \mathcal{M}_O^{-1}\mathcal{M}_I), \tag{33}$$

we see that the first term contains only the background field A and thus will not contribute to diagrams with external χ's and K_μ's. Expanding the second term in a series, one finds that only the quadratic term contains factors of $\chi K_\mu A_\nu$ and thus contributes to the relevant three-point functions. Thus

$$\Gamma^{(1)}_{\chi K_\mu}[A] = \frac{1}{4}\operatorname{str}(\mathcal{M}_0^{-1}\mathcal{M}_I)^2. \tag{34}$$
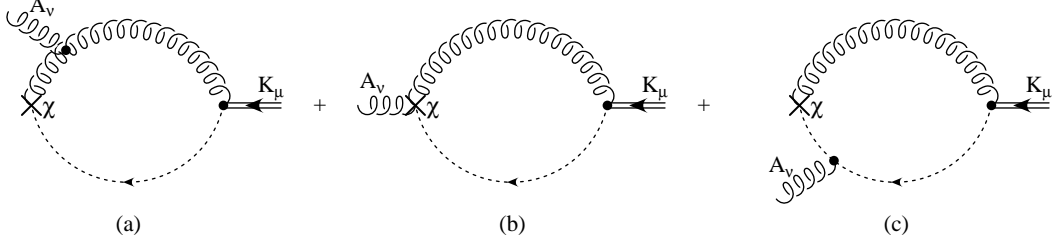
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FIG. 1. The one-loop contributions to $\Gamma_{\chi K_\mu A_\nu}$. (Curly and dashed lines indicate gauge bosons and ghosts respectively.)

Finding the required inverses and taking the supertrace we get that in momentum space $\Gamma^{(1)}_{\chi K_\mu A_\nu}$ is given by those terms in the expression

$$
\mathrm{tr}\frac{\chi}{2\alpha}\int\frac{d^3p\,d^3q}{(2\pi)^3}\left[\left(i\epsilon_{\kappa\beta\lambda}\frac{p_\beta}{p^2}-\alpha\frac{p_\kappa p_\lambda}{p^4}\right)-g\left(i\epsilon_{\kappa\beta\sigma}\frac{p_\beta}{p^2}-\alpha\frac{p_\kappa p_\sigma}{p^4}\right)\right.
$$
$$
\times\left(\frac{i}{\alpha}\left(p_\sigma A_\pi(-)+A_\sigma(-)p_\pi\right)+\epsilon_{\sigma\omega\pi}A_\omega(-)\right)\left.\left(i\epsilon_{\pi\delta\lambda}\frac{p_\delta}{p^2}-\alpha\frac{p_\pi p_\lambda}{p^4}\right)\right]
$$
$$
\left[ip_\lambda+gA_\lambda(-)\right]\left[\frac{-1}{p^2}-\frac{1}{p^2}ig\left(p_\mu A_\mu(-)+A_\mu(-)p_\mu\right)\frac{1}{p^2}\right]gK_\kappa(+) \tag{35}
$$

which are linear in A_ν.

One can readily see that upon expanding, the term with the fields A coming from the first, second and third set of square brackets correspond to the Feynman diagrams shown in Fig. 1 (a), (b), and (c) respectively. The remaining two point functions appearing in equation (26) are simply

$$
\frac{\delta^2\Gamma^{(0)}}{\delta A_\mu(x)\delta Q_\nu(y)}=\epsilon_{\mu\lambda\nu}\partial^y_\lambda\delta(y-x)\,. \tag{36}
$$

Upon using the expressions for $\Gamma^{(1)}_{AA}$, $\Gamma^{(1)}_{\chi K_\mu A}$ and $\Gamma^{(0)}_{AQ}$ from equations (14), (35) and (36) respectively, we see that the Nielsen identity of (26) is indeed satisfied to order g^2.

IV. TOPOLOGICAL OBSERVABLES

In the section II we derived the existence of gauge-parameter dependence in the two-point correlation $\langle V(x)V(y)\rangle$. Although these terms do not seem to have appeared before in the literature, we should not be too surprised by their existence since, as shown section III, this dependence is just a consequence of the BRS variation of operator $V(x)V(y)$.

We now turn our attention to the matter of gauge-parameter dependence in the topological invariants of the theory. Topological observables have been of primary interest in the development of Chern-Simons theory (eg. see [1,2,19]). It is well-understood that the conventional gauge-fixing procedure of Faddeev and Popov requires the introduction of a metric in the field dynamics and breaks, not only the local gauge invariance, but also the topological invariance of the theory. However, as argued in [20], the topological invariance

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can be shown to be recovered on the physical (BRS invariant) space. We show in this section that the interplay between gauge-fixing and topology may play another role.

The usual observables in Chern-Simons theory are the non-local metric-independent *Wilson link* operators whose expectation values are given by

$$\langle W[L] \rangle = \left\langle 0 \left| T \prod_{C_R \in L} W[C_R] \right| 0 \right\rangle , \tag{37}$$

where L can be any non-intersecting set of knots $\{C_R\}$, and

$$W[C_R] = \mathrm{tr}_R \mathcal{P} \exp \left\{ ig \oint_C dx^\mu \cdot V_\mu(x) \right\} \tag{38a}$$

$$\equiv \mathrm{tr}_R \sum_{n=0}^{\infty} (ig)^n \oint_C dx_n \cdot V(x_n) \int_x^{x_n} dx_{n-1} \cdot V(x_{n-1}) \cdots \int_x^{x_2} dx_1 \cdot V(x_1) \tag{38b}$$

is the *Wilson loop* operator of an oriented closed curve C (starting at an arbitrarily chosen point x) and associated Lie algebra representation R; $\mathcal{P}$ refers to "path ordering".

The fundamental problem which occurs in trying to identify the Wilson loop operators with topological gauge-independent observables arises from the regions of the integrals in (38) where two or more of the gauge fields occur at a coincident point. It is well known in quantum field theory that composite quantum fields require special regularization over and above any regularization of the elementary fields (see for example Collins [21]). Although, these coincident contributions do not introduce any further divergences into the expectation value of $W[C]$ (owing to the zero-measure of the region over which they contribute) they do serve to destroy the topological properties which are enjoyed by $W[C]$ at the classical level. It is thus found that, while the expectation values of cross-terms between the various elements of the link in (37) have an *a priori* well-defined topological meaning, the *self-interacting* terms (those involving more than one power on the gauge-connection on any given knot) do not.

The solution which is typically proposed is as follows. Instead of thinking of a given knot C_R as a 1-manifold, we can allow them to have a slightly extended structure: that of a ribbon, $\tilde{C}_R$ as shown in figure 2. This construction (referred to as *framing*) has been used extensively [19,2] to give topological meaning to the Wilson line operator which will thus depend on the topological structure of the ribbon.
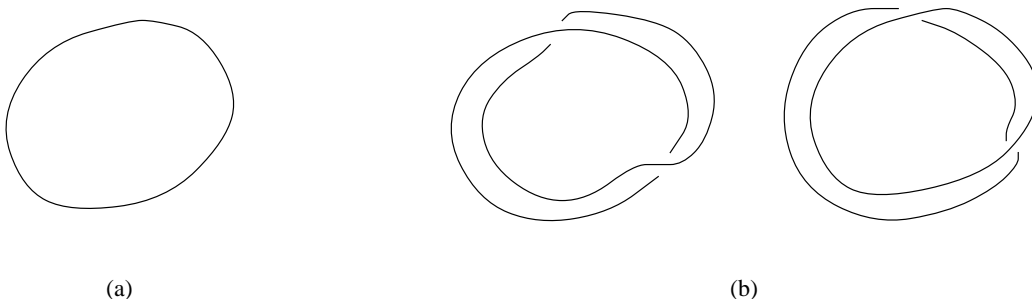

(a)   (b)

FIG. 2. The unknot (a) is shown here together with two topologically distinct framings (b).

At this point we turn our attention to the leading α dependence of the two-point contribution to the expectation value (37) resulting from tree-level effects and the radiative
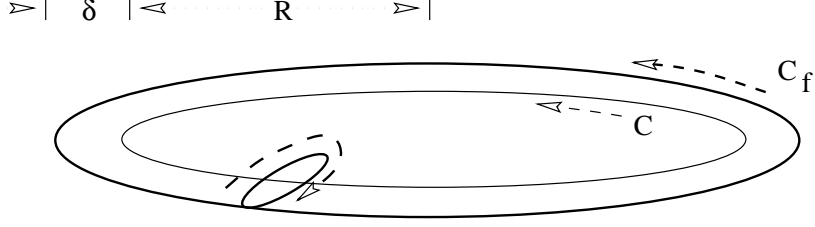
FIG. 3. Linked framing for a circular unknot.

correction given by (14). For this we shall content ourselves with a trivial link L composed, for example, of only a single circular unknot $\{C_R\}$. Since, for the moment, we are concerned only with the $n = 2$ term of (38b) it will be sufficient to make a simple implementation of the framing procedure given by

$$T^{(2)}[C, C_f]_R = \mathrm{tr}_R \frac{(ig)^2}{2!} \oint_C dx^\mu \oint_{C_f} dy^\nu \langle 0|V_\mu(x)V_\nu(y)|0\rangle \ . \tag{39}$$

where the framing $\tilde{C}$ has been reduced to two curves C (the original knot) and C_f (see, for instance, figure 3). $T^{(2)}$ refers to the contribution to $\langle W[L]\rangle$ in (37) coming from a single Wilson loop C with framing C_f that is of second order in the vector field.

Now, the propagator for Chern-Simons theory is, from (1) and (2), proportional to

$$\Lambda_{\mu\nu}^{ab} = \left[\frac{i\epsilon_{\mu\lambda\nu}p_\lambda}{p^2} - \alpha\frac{p_\mu p_\nu}{p^4} \right] \delta^{ab} \ . \tag{40}$$

Let us first consider the contribution of the term in (40) of order α to (39). The tree-level two-point function in coordinate space is just the Fourier transform of (40). From invariance arguments we must have

$$\int d^3p \frac{p_\alpha p_\beta}{p^4} e^{i\overline{p}\cdot\overline{r}} = A\frac{\delta_{\alpha\beta}}{r} + B\frac{r_\alpha r_\beta}{r^3} \ , \tag{41}$$

and thus by contracting (41) with $\delta_{\alpha\beta}$ and $r_\alpha r_\beta$ we obtain

$$\frac{3A + B}{r} = \frac{4\pi}{r} \int_0^\infty dx \frac{\sin x}{x} = \frac{2\pi^2}{r} \ , \tag{42a}$$

$$(A + B)r = 4\pi r \int_0^\infty dx \left[\frac{\sin x}{x} + 2\left(\frac{\cos x}{x^2} - \frac{\sin x}{x^3} \right) \right]$$
$$= 0 \ , \tag{42b}$$

so that $A = -B = \pi^2$. Hence, by (40 - 42), to order α, and to zero-loop order in the loop expansion, (39) is proportional to the integral $I^{(0)}$ given by

$$I^{(0)} = \pi^2 \oint_C dx_\lambda \oint_{C_f} dy_\sigma \left[\frac{\delta_{\lambda\sigma}}{|\overline{x} - \overline{y}|} - \frac{(x - y)_\lambda (x - y)_\sigma}{|\overline{x} - \overline{y}|^3} \right]$$
$$= \pi^2 \oint_C dx_\lambda \oint_{C_f} dy_\sigma \frac{\partial^2}{\partial x^\lambda \partial x^\sigma} |\overline{x} - \overline{y}| = 0 \ . \tag{43}$$

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Since, in (43) we have an integral around a closed loop of a gradient, it vanishes. Thus, to the order we have considered, there is no α dependence in $\langle W[C] \rangle$, as defined above. We now consider the effect of including the result of (14) on $T^{(2)}$ in (39).

To order α, the one-loop contribution to (39) involves having to consider, by (14) and (40),

$$\left(\frac{i\epsilon_{\mu\lambda\alpha}p_\lambda}{p^2} \right) \left(\frac{i\alpha\epsilon_{\alpha\sigma\beta}p_\sigma}{\sqrt{p^2}} \right) \left(\frac{i\epsilon_{\beta\kappa\nu}p_\kappa}{p^2} \right) = \frac{i\alpha\epsilon_{\mu\lambda\nu}p_\lambda}{(p^2)^{3/2}} . \tag{44}$$

Since we must have

$$\int d^3p \frac{p^\lambda e^{i\overline{p}\cdot\overline{r}}}{p^3} = f \frac{r^\lambda}{r^2} , \tag{45}$$

we must have, upon contracting (45) with r^λ,

$$f = -4\pi i \int_0^\infty dx \left(\frac{x\cos x - \sin x}{x^2} \right)$$
$$= 4\pi i . \tag{46}$$

Consequently, by (44) and (46), we see that the one-loop contribution to the α^1 term in (39) involves having to determine

$$I^{(1)} = -4\pi\alpha \oint_C dx^\lambda \oint_{C_f} dy^\sigma \frac{\epsilon_{\lambda\kappa\sigma} (x - y)_\kappa}{|\overline{x} - \overline{y}|^2}$$
$$= -4\pi\alpha \oint_C dx^\lambda \oint_{C_f} dy^\sigma \epsilon_{\lambda\kappa\sigma} \frac{\partial}{\partial x^\kappa} \ln |\overline{x} - \overline{y}| , \tag{47a}$$

$$= -4\pi\alpha \int\!\!\int_S dS^\lambda \oint_{C_f} dy^\sigma \left(\frac{\partial}{\partial x^\lambda} \frac{\partial}{\partial x^\kappa} - \delta_{\lambda\kappa} \frac{\partial^2}{\partial x^2} \right) \ln |\overline{x} - \overline{y}| , \tag{47b}$$

where S is the surface enclosed by C. The first integral in (47b) vanishes as it involves the integral of a total derivative around the closed curve C_f, the second disappears if dS^λ is normal to dy_σ.

We note that, upon applying Stokes' theorem to each of the line integrals in (47a), our equation becomes

$$I^{(1)} = -8\pi\alpha\epsilon_{\lambda\kappa\sigma} \int\!\!\int_S dS^x_\lambda \int\!\!\int_{S_f} dS^y_\kappa \frac{(x - y)_\sigma}{|\overline{x} - \overline{y}|^4} , \tag{47c}$$

where S and S_f are two dimensional surfaces bounded by C and C_f respectively.

For the circle depicted in figure 3, and its framing, the integral of (47a) reduces to

$$I^{(1)} = 4\pi\alpha R\delta \int_0^{2\pi} d\theta \int_0^{2\pi} d\phi \frac{R(1 - \cos\theta)\cos\phi - \delta\cos\theta}{2R^2 + \delta^2 - 2R^2\cos\theta + 2\delta R\cos\phi(1 - \cos\theta)} , \tag{48a}$$

which, evaluating the integral over θ, becomes

$$= 4\pi\alpha R\delta \left\{ \frac{2\pi^2}{\delta} \left(1 - \sqrt{1 - \frac{\delta^2}{R^2}} \right) - \frac{\pi}{R} \int_0^{2\pi} d\phi \frac{2R^2 + 3R\delta\cos\phi + \delta^2}{(R + \delta\cos\phi)\sqrt{4R^2 + 4\delta R\cos\phi + \delta^2}} \right\} , \tag{48b}$$

provided $\delta \neq 0$. We see that this vanishes as $\delta \to 0$, showing that this contribution to the dependence on α disappears as the width of the frame shrinks to zero.

The implementation of the framing procedure varies among the various authors. The method described above is reasonable when one is concerned with only the nth term in the expansion (38b) since in that case one considers the framing to be composed of n "parallel" curves; however it is not applicable when considering the expansion as a whole. Hereafter we will adopt a more general interpretation of simply distributing the "topological current" along the cross section of an appropriate set of ribbons; then only at the end of the calculation should we reduce the ribbons to the original knots through the appropriate limiting procedure. We parameterize the frame of a knot C by

$$\tilde{C} : \; x^\mu = x_C^\mu(\sigma, t) \quad \text{where } \sigma \in [0, 1], \; t \in [0, 1], \tag{49}$$

such that $x_C^\mu(\sigma, 0) = x_C^\mu(\sigma, 1)$, and $x_C^\mu(\sigma, t)$ is an orientable surface (thereby excluding such ribbons as the Möbius strip, which has but one edge.) Here, we have in mind that t should parameterize the length of the loop, while σ corresponds to integration along the cross-section of the ribbon. In order to make the framing of the knot (or, in general the link) topologically definite we should also impose the non-intersection requirement that $x_C^\mu(\sigma, t) = x_{C'}^\mu(\sigma', t')$ only if $\sigma = \sigma'$, $t = t'$ (modulus 1) and $C = C'$. Then when we come to regulate the topological invariants of the theory, we should replace (38) by the following *framed* Wilson loop:

$$W[\tilde{C}_R] = \text{tr}_R \mathcal{P} \exp \left\{ ig \int_0^1 d\sigma \int_0^1 dt \, \dot{x}_C^\mu(\sigma, t) V_\mu(x_C(\sigma, t)) \right\} \tag{50a}$$

$$\equiv \text{tr}_R \sum_{n=0}^\infty (ig)^n \int_0^1 d\sigma_n \int_0^1 dt_n \, \dot{x}_C(\sigma_n, t_n) \cdot V(x_C(\sigma_n, t_n))$$

$$\times \int_0^1 d\sigma_{n-1} \int_0^{t_n} dt_{n-1} \, \dot{x}_C(\sigma_{n-1}, t_{n-1}) \cdot V(x_C(\sigma_{n-1}, t_{n-1})) \tag{50b}$$

$$\vdots$$

$$\times \int_0^1 d\sigma_1 \int_0^{t_2} dt_1 \, \dot{x}_C(\sigma_1, t_1) \cdot V(x_C(\sigma_1, t_1)) \; .$$

We will eventually be interested in reducing the "width" of the above framing to zero in order to coincide with the original knot. The appropriate limiting procedure in this case is to replace $\tilde{C}_R$ by $\tilde{C}_R^\epsilon : x^\mu = x^\mu(\epsilon\sigma, t)$, and then take the simple limit $\epsilon \to 0$ which has the property that $\tilde{C}_R^\epsilon \to C_R$.

Now the main point of this section concerns the effects that the framing procedure has on the gauge invariance of the theory. The Wilson loop operator and the pure Chern-Simons action are both well known to be invariant under the infinitesimal gauge transformation

$$V_\mu(x) \to V_\mu(x) - \tfrac{1}{g} \partial_\mu \omega(x) + i[V_\mu(x), \omega(x)], \tag{51}$$

however it can be proven that the framing procedure of equation (50) is *not*. In fact, the framed loop operator transforms according to $W[\tilde{C}_R] \to W[\tilde{C}_R] + \delta W[\tilde{C}_R]$ where

$$\delta W[\tilde{C}_R] = -g \int_0^1 d\sigma \int_0^1 d\sigma' \int_0^1 dt \, \dot{x}^\mu(\sigma, t) \text{tr}_R W_{x_C(t)}^{x_C(t)}[\tilde{C}_R] \tag{52}$$

$$\times \Big[V_\mu(x_C(\sigma, t)), \omega(x_C(\sigma, t)) - \omega(x_C(\sigma', t)) \Big].$$

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Here $W_{x_C(t)}^{x_C(t)}[\tilde{C}_R]$ is the R representation of the framed *Wilson line* parallel transport operator $\mathcal{P} \exp\left[ig \int_0^1 d\sigma \int_0^1 dt \dot{x}_C^\mu(\sigma,t) V_\mu(x_C(\sigma,t))\right]$ along the closed curve C terminating at the point $x_C(t)$. It is clear that $\lim_{\epsilon \to 0} \delta W[\tilde{C}_R^\epsilon]$ vanishes at the classical level. At the quantum level, equation (52), along with the Nielsen transformation of equation (18) (in the absence of a background field) can be combined to give the following expression for the gauge parameter dependence of the framed Wilson loop:

$$\frac{\partial}{\partial\alpha}\langle W[\tilde{C}_R]\rangle = g^2 \int_0^1 d\sigma \int_0^1 d\sigma' \int_0^1 dt\, \dot{x}^\mu(\sigma,t) \tag{53}$$
$$\times \frac{\partial}{\partial\chi}\left\langle \mathrm{tr}_R W_{x_C(t)}^{x_C(t)}[\tilde{C}_R]\Big[V_\mu(x_C(\sigma,t)), c(x_C(\sigma,t)) - c(x_C(\sigma',t))\Big]\right\rangle .$$

In this latter equation $c(x)$ is the ghost field of the BRS multiplet and the vacuum expectation value $\frac{\partial}{\partial\chi}\langle\cdots\rangle$ refers to the matrix element in the presence of a single operator insertion $\frac{1}{2\alpha}\bar{c}\partial\cdot V$ (analogous to equation (17)). Equation (53) follows from taking $\frac{\partial}{\partial\chi}$ of (21) with vanishing background field provided that we identify Γ with the vacuum expectation value of the framed Wilson loop of (50). Whether $\frac{\partial}{\partial\alpha}\langle W[\tilde{C}_R^\epsilon]\rangle$ generally vanishes in the $\epsilon \to 0$ limit is unclear to us, but appears to be worth further consideration.

V. DISCUSSION

In this paper we have calculated part of the two-point function in pure non-Abelian Chern-Simons theory. The remaining parts of the two point function have been computed before, namely the α-independent contribution from the ghost Lagrangian to the modulus $|\Gamma^{(1)}|$ [14] and the contribution from the phase of $\Gamma^{(1)}$ [11].

The result which we have obtained clearly indicates that this two point function is *both* gauge *and* metric dependent. This is in contrast to the result in [1] where topological arguments were used to argue that radiative effects serve only to shift the coupling constant. The metric dependence of the two point function was noted previously [14] where the computation was carried out in the Landau-Honerkamp gauge (i.e. $\alpha = 0$) using operator regularization. The result of this paper is, to our knowledge, the first indication of gauge dependence in this two point function. The regularization procedure cannot be held responsible for this dependence as it has not been necessary to explicitly introduce any regulating parameter into the Lagrangian. Indeed, the dependence on the gauge parameter has been shown to be consistent with the Nielsen identities in section III, above.

The relationship between the vacuum expectation value of Wilson line operators and the N-point correlation functions has been explored in [2]. The discussion there is simplified as these authors, by using a regulating technique which does not respect BRS symmetry in the tree level effective action [6], find that there are no radiative corrections to the two and three point functions up to two-loop order (and hence that radiative corrections do not shift the coupling constant, as anticipated in [1]). However, the effect of including the radiative corrections found in [11,14] and in equation (14) above on the vacuum expectations value of framed Wilson line operators has, in the preceding section, been shown to give a possible contribution of order α. This is clearly a problem worth pursuing.

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We note that operator regularization, which is very similar to the techniques employed above, has been used to demonstrate the absence of any need to renormalize Chern-Simons theory at two-loop order when $\alpha = 0$ [22].

VI. ACKNOWLEDGEMENTS

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